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传递矩阵法特征值问题解法改进

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一、实、复特征值求解过程的改进

传递矩阵法实特征值方程的形式为:

$$D(\Omega) = 0 \tag{1}$$

式中 D (Ω) 是随Ω而变化的 N 阶特征行列式。经过三角化处理后, D (Ω) 的值为

$$D(\Omega) = \prod_{j=1}^{N} d_{jj}(\Omega) \tag{2}$$

其中 $d_{jj}(\Omega)$ 为三角化后行列式 $D(\Omega)$ 的主对角元素。若对于两个不同的自变量Ω_a和Ω_b, 有

$$D(\Omega_a) \cdot D(\Omega_b) < 0 \tag{3}$$

则在区间$\Omega_a \sim \Omega_b$中, 必然至少存在一个Ω值满足方程 (1)。若采用割线法求解方程 (1), 则利用 D (Ω_a) 和 D (Ω_b) 的值可得Ω的逼近值为

$$\Omega_c = \Omega_a + (\Omega_b - \Omega_a)/(1 - D(\Omega_b)/D(\Omega_a)) \tag{4}$$

事实上, 由于计算机的浮点数表达范围有限, 因此直接利用式 (2) 计算特征行列式值时, 常易遇到因行列式的绝对值过大或过小而引起数值溢出, 导致求解失败。为解决这一问题, 令

$$a_{jj}(\Omega) = d_{jj}(\Omega) \cdot |d_{jj}(\Omega)|^{1/k} / |d_{jj}(\Omega)| \tag{5}$$

$$A(\Omega) = \prod_{j=1}^{N} a_{jj}(\Omega) \tag{6}$$

由式 (5) 和式 (6) 可知, 当 k 大于 1.0 时, A (Ω) 的绝对值的变化范围将大大小于 D (Ω) 的绝对值的变化范围。因此, 若用 A (Ω) 作为特征值方程求解时的寻根依据, 并对式 (4) 进行相应的修改, 即有可能在完全保留割线法原有的逼近特性的基础上, 根本排除方程求解过程中发生数值溢出的可能性。

利用式 (2)、(4)、(5)、(6) 得

$$\Omega_c = \Omega_a + (\Omega_b - \Omega_a)/(1 - R) \tag{7}$$

其中

$$R = (A(\Omega_b)/A(\Omega_a)) \cdot |A(\Omega_b)/A(\Omega_a)|^{k-1}$$

式 (7) 不仅保留了式 (4) 的逼近特性, 同时由式 (5) 和式 (6) 可知, 只要 k 值足够大, 利用式 (7) 求解方程 (1) 时即可完全避免数值溢出问题的出现。

传递矩阵法复特征值方程的形式为:

$$D(\lambda) = 0 \tag{8}$$

式中, 特征变量λ及特征行列式 D (λ) 的值均为复数, 经三角化处理后 D (λ) 的值为

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$$D(\lambda) = \prod_{j=1}^{N} d_{jj}(\lambda) \tag{9}$$

若采用切线法求解 (8)，并以复平面上某一点λ_1作为寻根起始点，则λ的逼近解可表达为

$$\lambda^* = \lambda_1 + h/(1 - D(\lambda_1 + h)/D(\lambda_1)) \tag{10}$$

其中 h 为求导步长。显然，在计算机上直接利用式 (9) 计算特征行列式 D (λ) 的值时，仍很容易遇到数值溢出问题。为此，令

$$a_{jj}(\lambda) = d_{jj}(\lambda)^{1/k} \quad , \quad A(\lambda) = \prod_{j=1}^{N} a_{jj}(\lambda) \tag{11}$$

由式 (9)、(10)、(11) 可得

$$\lambda^* = \lambda_1 + h/[1 - (A(\lambda_1 + h)/A(\lambda_1))^k] \tag{12}$$

在 k 大于 1.0 的情况下，由式 (9) 和式 (11) 可知，A (λ) 的模数的变化范围大大小于 D (λ) 的模数变化范围。故此利用式 (12) 来求解方程 (8) 时，只要 k 值足够大，求解过程中将完全不再出现数值溢出问题。

二、算例分析与讨论

为验证所述改进方法的先进性，本文分别采用原有方法和改进方法对某实际发动机的转子 —支承 —机匣系统进行了实、复数特征值分析和比较，结果简述如下：

当采用改进的割线法 (即式 (7)，取 k = 10) 对系统进行实特征值分析时，在求根区间 Ω_a = 1000 1/ s、Ω_b = 1200 1/ s 中仅经三次逼近即得特征根 Ω = 1171.2 1/ s，精度为 10^{-5}。在求解过程中式 (6) 中 A (Ω) 的绝对值最大仅达 8.72×10^{8}。而直接采用割线法求解同一问题时，一开始即遇到计算机数值上溢问题，直至将计算模型的自由度数减至原来的 30 % (即将节点个数由 108 减至 32) 时方消除溢出问题，且式 (2) 中 D (Ω) 的绝对值最大时高达 7.86×10^{35}。此外，直接采用割线法对简化模型进行分析时所得结果为 Ω = 1247.5 1/ s，同原有计算模型分析结果间的相对误差为 6.5 %。

在求解同一系统的复数特征值时，采用改进方法 (仍取 k = 10) 从 λ_1 = 0 + 0i 处出发经五次逼近后求得系统的一对共轭复特征值为 λ = - 5.7 ±2024.1i。而直接采用切线法求解时，在出发点处即遇到了数值溢出问题。被迫改用简单模型后，方求得与前述结果相对应的一对特征值为 λ = - 5.1 ±2182.6i。两结果间模数相对误差为 7.8 %。此外，采用改进方法时，式 (11) 中 A (λ) 的模最大仅达 6.16×10^{5}。而直接采用切线法分析简单模型时，式 (9) 中 D (λ) 的模数最大达 8.72×10^{31}。

需要指出，改进方法中所出现的 k 值在大于 1.0 的情况下总能明显地缩小寻根依据的变化幅度。但为使改进后的解法更可靠，k 值不可太靠近 1.0，应选用稍大的值 (例如 k = 10)。k 值选定后，一般情况下毋须调整，即可保证特征值求解过程中不再出现数值溢出问题。

参 考 文 献

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DIGITAL REAL-TIME SIMULATION OF A TWIN-SPOOL TURBOJET

Liu Shangming, Wang Xueyu, Zhu Xingjian
(Beijing University of Aeronautics and Astronautics)

ABSTRACT A digital real-time simulation by means of single micro-computer has been developed for an actual two-spool afterburning turbojet engine. In order to overcome the contradiction between the computation precision and the speed necessary to digital real-time simulation technique, the characteristics of engine components have been reasonably simplified and modified and 80286 macro-assembly, 80287 float arithmetic parallel processing method and some programing techniques have been adopted. A non-linear, large-deviation digital real-time simulation has been obtained. The results of this simulation are comparaed with the measured data. It is shown that its calculation accuracy and speed are suitable for the real-time simulation.

TECHNICAL NOTES

EXPERIMEATAL INVESTIGATION OF A SMALL TWO-STAGE TRANSONIC AXIAL COMPRESSOR

Li Liangming (Nanhua Powerplant Institute)

ABSTRACT A small-dimension two-stage transonic axial compressor has been designed with blade tip veloicty 436m/s and design revolution 45000r/min. Its tests has been carried out to investigate the performance of this compressor. It is shown that its performance attains or exceeds its design specifications. This paper also considers the effects of radial and axial clearances, probe blockage, step matching and measuring techniques on the performance of the compressor.

NUMERICAL STUDY OF END-WALL BOUNDARY LAYER AND BLADE FORCE DEFECT IN CASCADES

Wu Hu, Liu Songling, Chen Fuqun (Northwestern Polytechnic University)

ABSTRACT A detail research has been completed for calculation of end-wall boundary layers within an axial compressor cascade. A new calculation model for blade force defect is proposed, which is associated with development of primary flow and transverse flow. A unique feature of this method lies not only in capability to calculate the development of end-wall boundary layers inside the cascade passage without any assumption about the exit of cascade, but also in ability to take into account the influecnce of its tip clearance. The calculational results of three heavily loaded cascades show that this model is superior to other existing models.

IMPROVEMENT IN SOLUTION OF EIGENVALUES WITH TRANSFER MATRIX METHOD ON COMPUTER

Li Yan (Nanhua Powerplant Research Institute)

ABSTRACT A measure is taken based on secant method and tangent method to improve the computational method of eigenvalues with transfer matrix method on computer. In the new measure

the references for searching the eigenvalues are the first k-th powers of the eigen determinants (where k is larger than 1.0) instead of the determinants themselves, and the searching formulas of the existing method are modified correspondingly. As the range of the references for searching the eigenvalues is reduced considerably, the phenomenon of numerical overflow is eliminated forever.

J - INTEGRAL FOR2-D VARIABLE - THICKNESS PLATE

Hong Qilin (Beijing University of Aeronautics and Astronoutics)
Wang Ge (East - China Metallurgical Institute)

ABSTRACT The formula of a J-integral presented by J. R. Rice is extended to problem of 2-D variable-thickness plates. The extended J-integral formula is derived in terms of the path independent. Then the integral conservation is verified by using finite element method. At last, the calculation results for some variable thickness plates made of photoelastic material are compared with their photoelastic test data, and the differences between them are less than 4 %. It is proved that the extended J-integral formula for the 2-D variable-thickness plates is path independent and can be taken as a fracture pamarater for damage tolerance analysis. Furtheremore, this integral can be solved numerically by using 2-D general finite element programs. It is therefore of practical value for the fracture analysis of I mode cracked plate with thickness which does not vary too mueh.

COMBUSTION PROCESSES OF HYDROGEN IN A SUPERSONIC AIR FLOW

Yang Aikuo (Computing Technical Research Institute)
Liu Ling and Wang Hongji (Northwestern Polytechnic University)

ABSTRACT According to the implicit numerical integration method proposed by Tyson. J, an improved computer program for pseudo-one-dimensional flow of combustion kinetics is developed with the aid of a chemical kinectic model with 12-species and 31 reaction. Variations of temperature, prssure, Mach number and mass fraction of chemical species in a supersonic combustion model are studied in detail. The results of scramjet combustor conditions corresponding to flight Mach numbers 6, 10 and 15 show that chemical reaction rates decrease at rather low temperature and pressure at a combustor inlet and there is not enough time to ignite the mixture-gas in the combustor at very high Mach number, in result the performance limits of scramjet kinetic combustion occur.

CHARACTERISTICS OF DUAL SWIRLERS SPRAYER DEVICE

Chen Hong and Chin Jushan (Beijing University of Aeronautics and Astronautics)

ABSTRACT The spray characteristics of dual radial swirlers mixing-cup sprayer device have been investigated with the aid of Malvern laser granulometer and checked the differences of its spray characteristics from those of the pressure atomizing fuel nozzle. Two air flow proportions of the first swirler to the second one are 50 % :50 % and 30 % :70 %. The airflow velocity at the exit-section has been designed in the range 37 to 74m/s. In the tests the fuel pressure ranges from 2.94 to 4.41 $\times$ 10^5Pa. The test results show that in the case of reasonable design, the sprayer device is able to provide better spray characteristics than pressure atomizing fuel nozzle.